Design of a Rear Differential

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MAE 3191

Abstract:

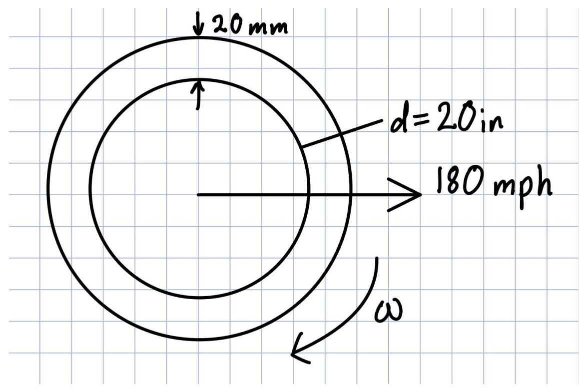
When a car makes a turn, the inner and outer wheels have to travel a different amount of distance. The outer wheel must travel faster or it will slip. A differential allows the wheels to turn at different speeds so that neither of the wheels slips when making a turn. Although the majority of modern cars use a more complicated type of differential known as a limited-slip differential, we chose to build an open differential because of its simplicity in design and concept.

Explanation of Loads:

A 2021 BMW M3 Competition sedan was chosen as the basis for this project. This car has a 3.0 liter M Twinpower Turbo inline 6 cylinder engine with an 8-speed M Steptronic automatic transmission. The engine produces 503 brake horsepower and 406 lb-ft of torque. The car is completely rear-wheel drive, so all the power is transferred to the rear two wheels. It is estimated that there is a 15% loss between brake horsepower (measured at the engine) and wheel horsepower (measured at the wheels). The horsepower at the rear wheel is 427.55 wheel horsepower. The max load on the differential is when the car is traveling at its maximum speed of 180 miles per hour. The car will be in 8th gear at this speed. The 8th gear gearing ratio of .64 and the final drive ratio of 3.15 were taken from the BMW website. The standard rear wheels have a diameter of 20 inches, and the rear tires have a sidewall that is 20 millimeters in height. Using this information we were able to find the angular velocity of the wheel attached to the shaft to be used for our output angular velocity. Using these specifications of the selected BMW model, the differential was designed.

Load Calculations:

Finding Total Wheel and Tire Diameter:



$$D = (20in \cdot \frac{1.ft}{12\ in}) + (20mm \cdot \frac{1ft}{304.8mm})$$

$$D = 1.73228ft$$

Angular Velocity of Wheel Calculation:

$$\omega = \frac{180mi}{1hour} \cdot \frac{5280ft}{1mi} \cdot \frac{1rev}{\pi \cdot 1.7322ft} \cdot \frac{1hour}{60min}$$

$$\omega_{wheels} = 2910.6255rpm$$

Torque Input from Drive Shaft at the Differential:

$$T_{gearbox} = i_{8thGR} \cdot T_{engine} = .64 \cdot 479lb \cdot ft \rightarrow T_{gearbox} = 306.56lb \cdot ft$$

Torque Output from Differential Final Drive Ratio:

$$T_{diff} = i_{finaldrive} \cdot T_{gearbox} = 3.15 \cdot 306.56lb \cdot ft \rightarrow T_{diff} = 965.664lb$$

Torque Output at the Differential to Each Axle:

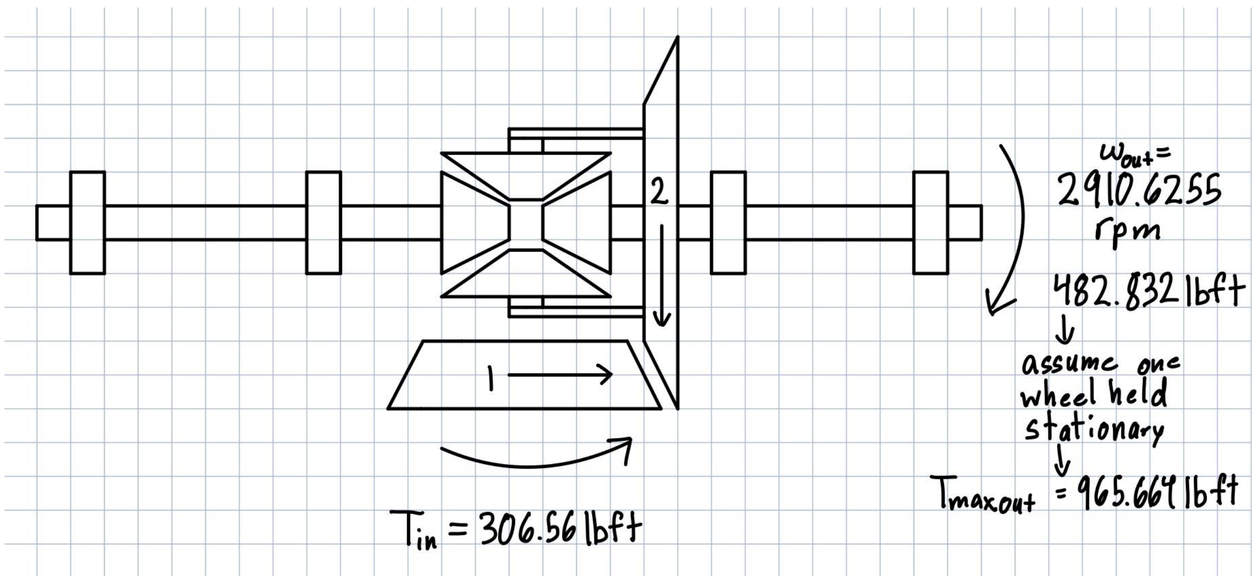
$$T_{LW} = T_{RW} = \frac{Tdiff}{2} = \frac{965.664lb \cdot ft}{2} \rightarrow T_{LW} = T_{RW} = 482.832lb \cdot ft$$

Brake Horsepower to Wheel Horsepower:

$whp = (1 - .15) \cdot bhp \rightarrow whp = 427.55hp$

Design:

Using the loads calculated a simple open differential was sketched to determine the constraints for the parts needed. The driveshaft was not considered part of the differential, so it is not included in the drawings.



Theoretical Input Angular Velocity (Angular Velocity of Gear 1) Calculation:

$$Gear\ Ratio = \frac{\omega_g}{\omega_p} \rightarrow 3.15 = \frac{2910.6255rpm}{\omega_p} \rightarrow \omega_p = 924.008rpm$$

Theoretical Gear Size Calculation:

Gear 2 was assumed to have a diametral pitch of 15 inches

$$Gear\ Ratio = \frac{d_g}{d_p} \rightarrow 3.15 = \frac{15in}{d_p} \rightarrow d_p = 4.761in$$

Based on our research the gears attached to the output shaft on other differentials, the gears attached to the shafts are usually ⅓ the size of the crown gear. Knowing that gear 2 has a 15-inch diametral pitch, it was determined gears with a 5-inch pitch would be used for the interior set of gears. Usually, these gears have a ratio of 1, so a special set of bevel gears with a gear ratio of 1 called miter gears were used.

The shaft diameter had to be determined with the given loads. In order to calculate this, it was decided that the material used for this shaft would be 1045 carbon steel. The yield stress of this steel was obtained from the internet to be 65300 psi. The diameter can be determined by using the equation for torsional shear stress (equation 4.3). The polar moment of inertia for a circular shaft can be found in appendix B1-A. the max shear stress must also be calculated using maximum shear stress theory and designing for a safety factor of 1. Knowing this, the diameter can be found.

Converting Torque to lb-in:

$$T = (965.664 lb \cdot ft) \cdot \frac{12in}{1ft} \rightarrow T = 11587.96 lb \cdot in$$

Finding Max Shear Stress:

$$FS = \frac{\sigma_{limit}}{2\tau_{max}} \rightarrow 1 = \frac{65300\frac{lb}{in^2}}{2\tau_{max}} \rightarrow \tau_{max} = 32650\frac{lb}{in^2}$$

Shaft Diameter Calculation:

$$\tau = \frac{Tr}{J}, J = \frac{\pi d^4}{32}, r = \frac{d}{2} \rightarrow \tau = (T \cdot \frac{d}{2})/(\frac{\pi d^4}{32})$$

$$32650\frac{lb}{in^2} = [(11587.96 lb \cdot in) \cdot \frac{d}{2}]/(\frac{\pi d^4}{32}) \rightarrow d_{shaft} = 1.218in$$

The shaft is rounded up to 2 inches to use an industry-standard size.

Redesign:

There are no pinion and crown gears with a gear ratio of 3.15 readily available, so the gear ratio was changed to 3 in order to use widely available parts. This changes the input angular velocity of the system as well as the input torque on the system. There are also no readily available gears with a 15-inch diametral pitch since this was an arbitrary starting point. It was decided to use the largest gears that could be found with a gear ratio of 3 in order to maximize room for the housing. The angular velocity of the pinion gear must be recalculated with the new gear ratio.

Actual Input Angular velocity (Angular Velocity of Gear 1) Calculation:

$$Gear\ Ratio = \frac{\omega_g}{\omega_p} \rightarrow 3 = \frac{2910.6255rpm}{\omega_p} \rightarrow \omega_p = 970.2085rpm$$

Actual Torque Input from Drive Shaft at the Differential:

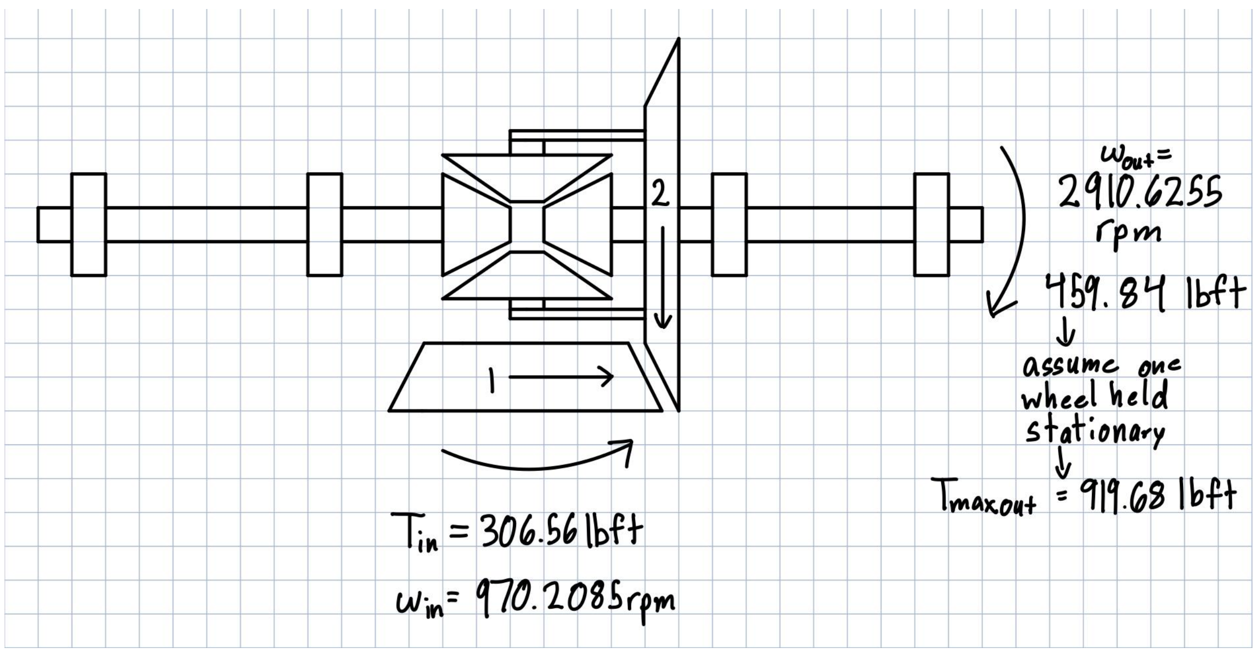
$$T_{diff} = i_{finaldrive} \cdot T_{gearbox} = 3 \cdot 306.56lb \cdot ft \rightarrow T_{diff} = 919.68lb \cdot ft$$

Actual Torque Output at the Differential to Axles:

$$T_{LW} = T_{RW} = \frac{Tdiff}{2} = \frac{919.68lb \cdot ft}{2} \rightarrow T_{LW} = T_{RW} = 459.84lb \cdot ft$$

But max torque load on the wheels occurs when one wheel is held stationary and the other wheel takes the full load. For analysis, the output torque at the shaft will be T=919.68lbft. The diameter of the output shafts does not need to be recalculated since the torque was lowered in the redesign.

The updated design is shown below



Part Selection:

After determining the parameters of the design parts were searched for on the internet. The part selection was based on design constraints and parts availability. Parts are sourced from KHK Gears and McMaster Carr.

Crown and Pinion Gear Selection

The most suitable gears were found on KHK gears. Given a needed gear ratio of 3, the largest gears with this ratio were surveyed in order to maximize room for the housing and interior gear assembly.

The first gears looked at were the KHK SB6-4515 crown gear paired with the SB6-1545 pinion. The crown and pinion gear have diametral pitches of 10.630 inches (270mm) and 3.543 inches (90mm) respectively. These are straight tooth steel bevel gear, with a 30 mm bore diameter. This set of gears was ruled out in order to minimize required machining that would have to be done to the gear due to the small diameter.

The second set of gears was the KHK SBS5-4515R crown gear and SBS5-1545L pinion. These are carbon steel spiral bevel gears. This set of gears has smaller diametral pitches than other gears available, and would also require more machining to accommodate the design of the differential.

The gears chosen are the KHK MBSA6-4515R crown gear and MBSA6-1545L pinion gear. These gears have the largest diametral pitches available on KHK at 10.630 inches (270mm) and 3.543 inches (90mm) respectively, allowing for maximum room for the design of the cage. The crown gear has a bore diameter of 4.33 inches (110mm) which allows for the shaft to pass through it without contact. These gears are made of SCM415 alloy steel, which has similar tensile and yield strength to the S45C steel used in the other two sets of gears. Additionally, the spiral gear and pinion design were best suited to handle increased loads at higher speeds, because the teeth stay in contact for longer compared to the straight toothed bevel gear and pinion.

Miter Gear Selection

Knowing that the set of internal gears should have a gear ratio of 1 and that the accepted practice is to choose a gear about ⅓ the size of the crown gear, the search began for a set of miter gears with a 3.543 inch (90mm) diametral pitch. The gears were found on KHK. There were several gears that could be used.

The first gear looked at was the MMS3-30L. This is a set of steel spiral miter gears. This was the lightest out of the gears looked at. This gear could have been suitable for the design.

The second gear looked at was the MMSG3-30L. This set of gears is similar to the last, also being a spiral miter gear made out of SCM415 alloy steel. This gear is thinner than the other one, which could have been advantageous from a design standpoint.

The gears chosen were the KHK MM3-30 gears. These are straight teeth miter gears also made of SCM415 alloy steel. These gears were chosen due to the simplicity of a straight-toothed gear, as well as having very similar properties to the other two gears looked at.

Bearing Selection

Assuming only the bearing closest to the differential takes thrust, a bearing that was rated for radial and thrust loads had to be chosen for this design. McMaster Carr was used to find the bearings. Knowing that the system is subject to large loads, roller bearings were chosen over ball bearings due to their increased rigidity and load capacity. The shafts necessary were calculated to have a 2-inch diameter, so the bearings chosen needed this inner diameter. With these requirements, there were several bearings that fit the requirements. The list was further cut down after excluding mounted bearings.

The three bearings that remained were tapered rolling bearings of the same manufacturer. The bearings with the smallest outer diameter were chosen in order to maximize space. All three bearings were suited to handle the given loads. The LM104949 tempered-roller bearing was chosen to be used in this differential.

Material Selection

1045 Carbon steel was chosen as the material for the shaft and housing due to its high yield strength of 70000-100000 psi. It is often used in the design of mechanical systems. This material can be heat treated to gain desired surface properties.

McMaster Carr sells high-strength 1045 carbon steel rods with a diameter of 3 inches. This can be machined down to the 2 inches required for this design, but the 3-inch rod was ultimately chosen so that further design could be done to optimize the shaft for this design. Optimizing the shaft was not able to be done within the time limit for the project.

For the housing and center pin, McMaster Carr sells sheets and bars of 1045 carbon steel. The material can be machined to make the housing. The required amount of material and sizes of material to fabricate the housing was not determined, since this project only called for the design of a gear train.

Fastener Selection

Three fasteners were required to put the model together. All are sold on McMaster Carr. Fasteners were primarily needed to assemble the housing onto the crown gear. They were selected based on size requirements.

The first fastener chosen was black-oxide alloy steel socket head screws with a length of 2.5 inches and an 8-32 thread size. This screw is used to fasten the housing on the front of the crown gear. A socket head was chosen so that the screws could be assembled flush with the back face of the crown gear. The size was determined based on the constraints of the top and bottom plates that hold the gears in the housing. 2.5 inches was the longest screw that could be used without interfering with the center pin.

The second fastener chosen was also a black-oxide alloy steel socket head screw but is 1 inch long with ¼”-28 threads. These fasteners are to fasten the top plate to the bearing carrier in front of the crown gear. The socket head was again chosen for a flush fit with the plate. 1 inch is the largest screw that can be used without drilling too far into the bearing carrier.

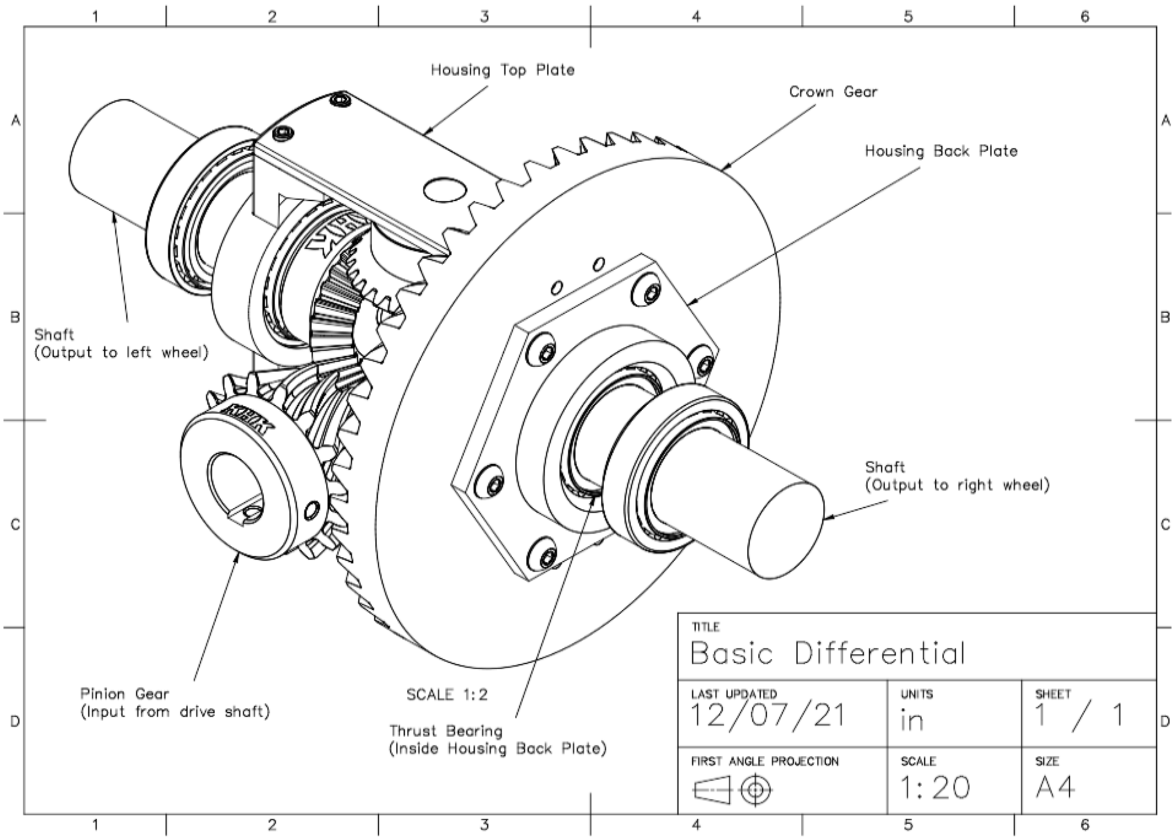
The last fastener needed was to secure the bearing plate to the back of the crown gear. The crown gear has 6 M10 threads spaced evenly on the back of the crown gear. After designing the plate, the needed length of the fastener could be determined. A bolt with a length of 0.866 inches (22mm) was needed. A button head screw was chosen since the plate is not thick enough to have socket head screws fit flush into the plate. The screw chosen is an 18-8 stainless steel button head hex drive screw, with a black-oxide coating, M10 x 1.50mm thread, with a length of 22mm.

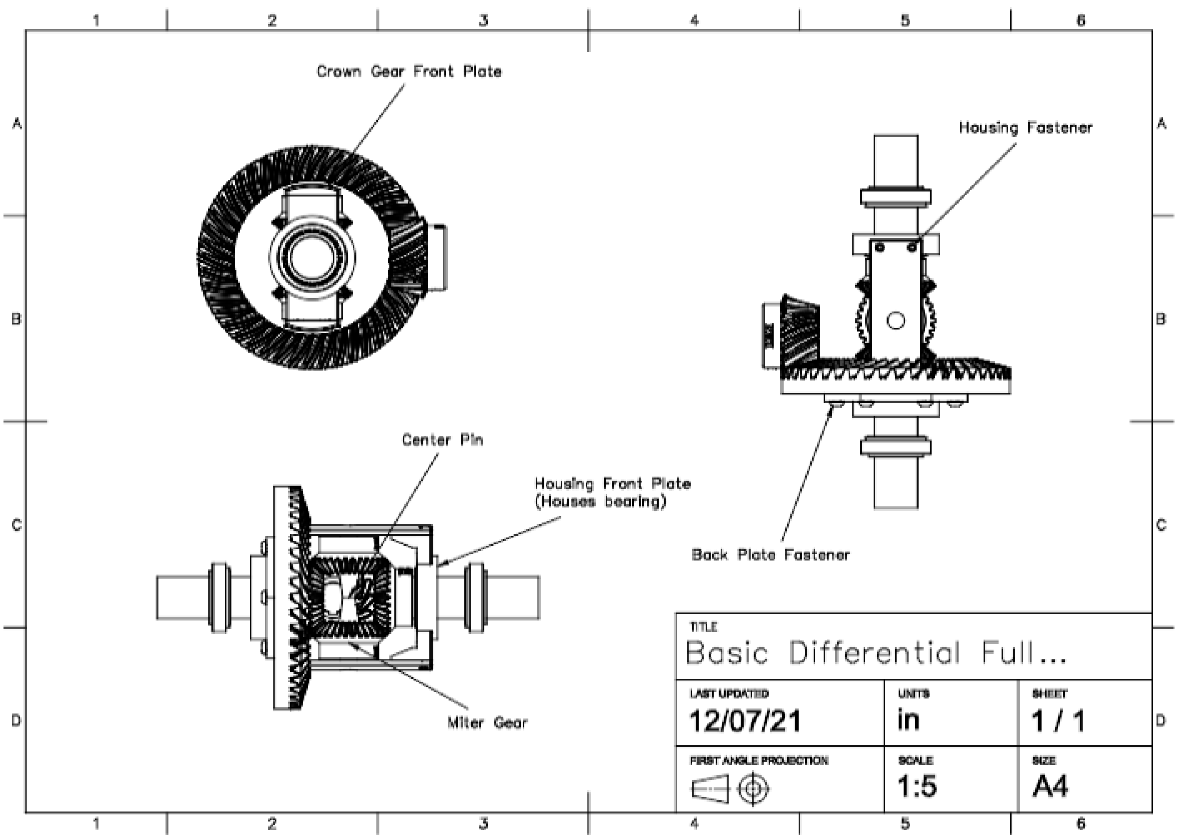
Bill of Materials:

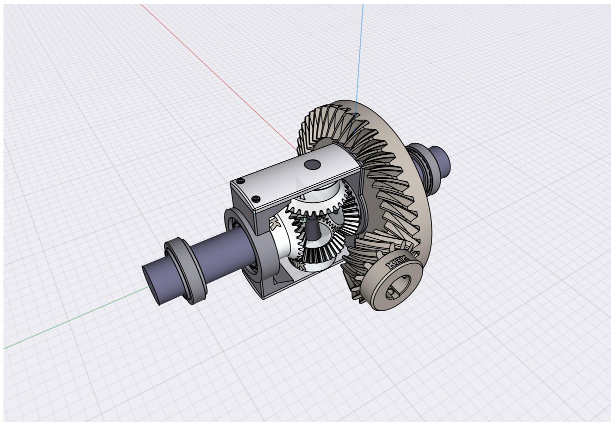
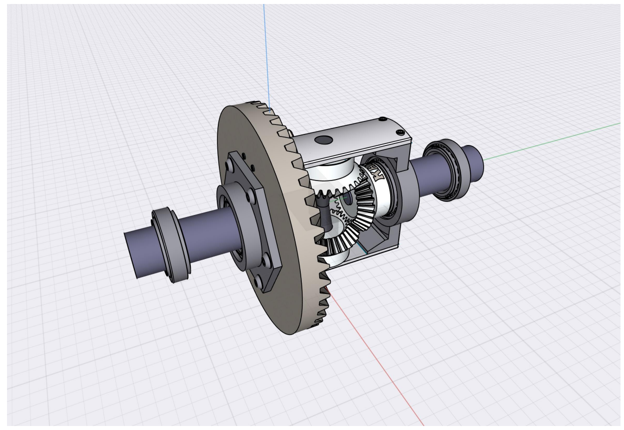
Part	Part Type	Vendor	Part #	Quantity	Price
Pinion Gear	Bevel Gear	KHK	MBSA6-1545L	1	$230.22
Crown Gear	Bevel Gear	KHK	MBSA6-4515R	1	$724.13
Miter Gears	Bevel Gear	KHk	MM3-30	4	$394.40
Taper-Roller Bearings	Bearing	McMaster Carr	LM104949	4	$201.24
Black-Oxide Steel Head Screw	Fastener	McMaster Carr	91251A442	4	$14.46
Black-Oxide Steel Head Screw	Fastener	McMaster Carr	90044A514	4	$14.45
Steel Button Head Screw	Fastener	McMaster Carr	97763A455	6	$16.02
1045 Carbon Steel Rod	Raw Material	McMaster Carr	8924k78	1	$162.17
1045 Carbon Steel	Raw Material				Quote Needed
				Total:	$1757.47

Final Design:

The CAD file was assembled in Shapr3D. The models were imported as STEP (.stp) files.


Housing Top Plate
Crown Gear
Housing Back Plate
Shaft
(Output to left wheel)
Shaft
(Output to right wheel)
Pinion Gear
(Input from drive shaft)
SCALE 1:2
Thrust Bearing
(Inside Housing Back Plate)
TITLE
Basic Differential
LAST UPDATED
12/07/21
UNITS
in
SHEET
1 / 1
FIRST ANGLE PROJECTION
SCALE
1:20
SIZE
A4


Crown Gear Front Plate
Housing Fastener
Center Pin
Housing Front Plate
(Houses bearing)
Back Plate Fastener
Miter Gear
TITLE
Basic Differential Full...
LAST UPDATED
12/07/21
UNITS
in
SHEET
1 / 1
FIRST ANGLE PROJECTION
SCALE
1:5
SIZE
A4

Analysis:

Three parts were selected for analysis of the model. The crown gear, a shaft, and a bearing. Analysis was done using AGMA equations from the textbook to verify the differential design. Factors were picked based on knowledge and assumptions of the differential system. Calculations were largely done in SI units and converted to British Standard units due to manufacturers largely designing things in the metric system.

Assumptions:

- Components have a required lifespan of 10^6 cycles
- Components don’t exceed 160 degrees Fahrenheit
- Components have at least 99% reliability
- The system is subject to moderate shock
- The highest stresses occur when one wheel is held stationary, all power is transmitted to the other wheel
- The casing is designed for accurate gear mounting for minimum deflection
- The system is subject to moderate shock loading
- Reaction torques from rear axles are neglected
- The bearings and gears are mounted directly next to each other

Crown Gear Analysis (Done by Omar, checked by Oscar)

The crown Gear was analyzed using three types of analysis: gear-tooth bending strength analysis, gear-tooth fatigue analysis, and surface fatigue analysis.

Gear-Tooth Bending Stress Analysis:

The average diameter and average pitch line velocity must be determined first to calculate the tangential force. These equations are for calculations using British Standard units.

$$d_{av} = d - b \sin \gamma \quad (16.18)$$

$$V_{av} = \pi d_{av} n \quad (16.19a)$$

$$F_t = 33{,}000\dot{W}/V_{av} \quad (16.20a)$$

Where d_{av} is in feet, V_{av} is in ft/min, n is in rpm, F_t is in-lbs, and W is in horsepower.

Average Diameter Calculation:

$$d_{av} = 270mm - (42mm)sin(72) \rightarrow d_{av} = 230.055mm \cdot \frac{1in}{25.4mm} \cdot \frac{1ft}{12in}$$

$$d_{av} = .75477ft$$

Average PitchLine Velocity Calculation:

$$V_{av} = \pi \cdot (.75477ft) \cdot (2910.62rpm) \rightarrow V_{av} = 6901.603\frac{ft}{min}$$

Tangential Force Calculation:

$$F_t = (33000 \cdot 427.55hp)/(6901.603\frac{ft}{min}) \rightarrow F_t = 2044.32lbs$$

Now, the Lewis Equation can be used in the following form. Factors were picked using charts from the book. The geometry factor (J) was determined using figure 16.14 to be .25. The velocity factor was found by finding the pitch line velocity of the gear using equation 15.13, then using figure 15.24 and the equations that come along with it. Since it was assumed that the system is subject to moderate shock, the overload factor was determined using table 15.1 to be 1.25. The gears are overhung, using table 16.1 the mounting factor was determined to be 1.5.

$$\sigma = \frac{F_t P}{bJ} K_v K_o K_m \qquad (15.17)$$

where

F_t	=	tangential load in pounds, from Eq. 16.20
P	=	diametral pitch at the large end of the tooth
b	=	face width in inches (should be in accordance with Eq. 16.17)
J	=	geometry factor from Figure 16.13 (straight bevel) or Figure 16.14 (spiral bevel)[1]
K_v	=	velocity factor. (When better information is not available, use a value between unity and curve C of Figure 15.24, depending on the degree of manufacturing precision.)
K_o	=	overload factor, from Table 15.1
K_m	=	mounting factor, depending on whether gears are straddle-mounted (between two bearings) or overhung (outboard of both bearings) and on the degree of mounting

Diameter Conversion to Feet:

$$d = 270mm \cdot \frac{1in}{25.4mm} \cdot \frac{1ft}{12im} \rightarrow d = .88582ft$$

Pitch Line Velocity Calculation:

$$V = \frac{\pi dn}{12} \rightarrow V = \frac{\pi(.88582ft)(2910.62rpm)}{12} \rightarrow V = 674.998\frac{ft}{min}$$

Velocity Factor Calculation:

Assuming precision shaved and ground gears

$$K_v = \frac{50+\sqrt{V}}{50} \rightarrow K_v = \frac{50+\sqrt{674.998\frac{ft}{min}}}{50} \rightarrow K_v = 1.52$$

Calculation of Gear-Tooth Bending Stress:

$$\sigma = \frac{(2044.32lbs)(10.630in)}{(1.654in)(.25)} \cdot 1.52 \cdot 1.25 \cdot 1.5 \rightarrow \sigma = 149779.19psi \rightarrow \sigma = 150$$

Conversion to MPa:

$$150ksi \cdot \frac{6.89476MPa}{1ksi} = 1034.21MPa$$

The bending fatigue strength must be calculated in order to determine the safety factor from equation 15.18. Knowing the ultimate strength of the steel is 655MPa and finding the factors from the book due to lack of better information, the gear-tooth bending fatigue strength can be calculated.

Gear-Tooth Bending Fatigue Strength Calculation:

$$S_n = S_n' C_L C_G C_S k_r k_t k_{ms} \quad (15.18)$$

where

S_n'	=	standard R. R. Moore endurance limit
C_L	=	load factor = 1.0 for bending loads
C_G	=	gradient factor = 1.0 for $P > 5$, and 0.85 for $P \le 5$
C_S	=	surface factor from Figure 8.13. Be sure that this pertains to the surface *in the fillet*, where a fatigue crack would likely start. (In the absence of specific information, assume this to be equivalent to a machined surface.)
k_r	=	reliability factor, C_R, determined from Figure 6.19. For convenience, values corresponding to an endurance limit standard deviation of 8% are given in Table 15.3.
k_t	=	temperature factor, C_T. For steel gears, use k_t = 1.0 if the temperature (usually estimated on the basis of lubricant temperature) is less than 160°F. If not, and in the absence of better information, use $$k_t = \frac{620}{460+T} \qquad (\text{for } T > 160^\circ\text{F}) \quad (15.19)$$
k_{ms}	=	mean stress factor. In accordance with Section 15.7, use 1.0 for idler gears (subjected to two-way bending) and 1.4 for input and output gears (one-way bending).

$S_n' = .5S_u \rightarrow S_n' = 327.5MPa,\ C_L = 1,\ C_g = 1,\ C_s = .75,\ k_r = .814,$
$k_t = 1,\ k_{ms} = 1.4$

$$S_n = (327.5MPa)(1)(1)(.75)(.814)(1.4) \rightarrow S_n = 279.91MPa$$

Safety Factor Calculation:

$$FS = \frac{S_n}{\sigma} \rightarrow FS = \frac{655MPa}{1034MPa} \rightarrow FS = .63$$

The crown gear will fail according to the gear tooth strength analysis since the calculated gear tooth bending stress of 1034.21 MPa is greater than the bending fatigue strength. The bending safety factor of this gear was still determined to be 0.63.

Bevel Gear Surface Fatigue Analysis

The bevel gear surface fatigue was calculated using equation 15.24. The same velocity, overload, and mounting factors are used from the gear-tooth bending strength analysis. The C_p value for bevel gears is 1.23 times the values found in table 15.4. From this table, the elastic coefficient of steel gears is $2829\sqrt{psi}$. The geometry I factor was calculated using equation 15.23. The gear tooth surface fatigue stress can be calculated using equation 15.25, then compared to determine the safety factor. The S-N curve for this steel could not be found, so values were determined using figure 15.27 (the tooth hardness is HRC 55, or 552, table 15.5 ($C_{Li} = 1.1$ for $10^6 cycles$), and table 15.6 ($C_R = 1$for 99% reliability).

Geometry Factor Calculation:

$$I = \frac{sin\phi cos\phi}{2} \cdot \frac{GR}{GR+1} \rightarrow I = \frac{sin(20)cos(20)}{2} \cdot \frac{3}{3+1} \rightarrow I = .1205$$

Surface Fatigue Calculation:

$$\sigma_H = C_p\sqrt{\frac{F_t}{bd_pI}K_vK_oK_m} \qquad (15.24)$$

$$\sigma_H = (2829\sqrt{psi})\sqrt{\frac{2044.32lbs}{(1.6535in)(10.6299in)(.1205)} \cdot 1.52 \cdot 1.25 \cdot 1.5}$$

$$\sigma_H = 7782258.71\sqrt{psi} \rightarrow \sigma_H = 2789.67psi$$

Conversion to MPa:

$$\sigma_H = 2789.67psi \cdot \frac{1MPa}{145.038psi} \rightarrow \sigma_H = 19.234MPa$$

Surface Fatigue Stress Calculation:

$$S_H = S_{fe} C_{Li} C_R \qquad (15.25)$$

$S_{fe} = 2.8(Bhn) - 69Mpa \rightarrow S_{fe} = 2.8(552) - 69Mpa \rightarrow S_{fe} = 1476.6M$

$S_H = (1476.6MPa)(1.1)(1) \rightarrow S_H = 1624.26MPa$

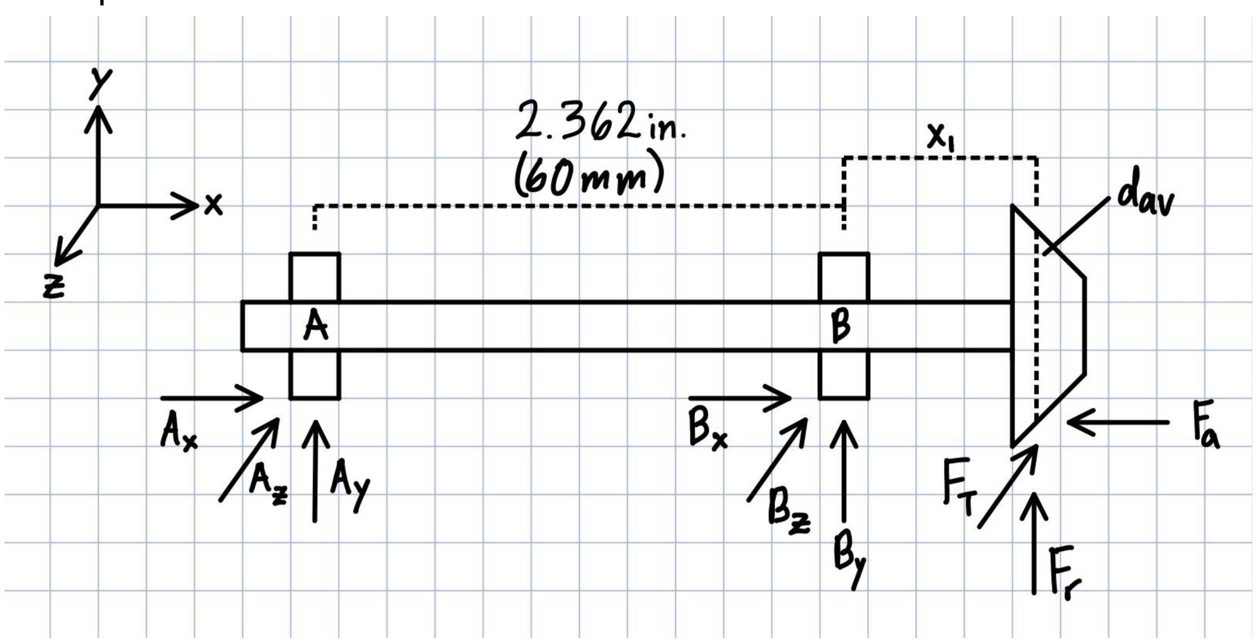
Safety Factor Calculation:

$$FS = \frac{S_H}{\sigma_H} \rightarrow FS = \frac{1624.26MPa}{19.234MPa} \rightarrow FS = 84.5$$

The gear tooth bending stress analysis demonstrates that the teeth from Gleason used on these gears are more than capable of handling the loads given, as shown by a safety factor 0f 84.5. The teeth will not fail for the given assumptions and loads.

Shaft Analysis (Done by Oscar, checked by Omar)

One of the output shafts was analyzed. A free body diagram was made and then analyzed. It is assumed that only bearing B takes axial forces. The shaft is rotating at 2910.62rpm, with 427.55hp and 919.68lb-ft of torque.



Average Diameter Calculation:

$$d_{av} = 90mm - (20)sin(45) \rightarrow d_{av} = 75.858mm \cdot \frac{1in}{25.4mm} \cdot \frac{1ft}{12in}$$

$$d_{av} = 2.986in = .2488ft$$

Average PitchLine Velocity Calculation:

$$V_{av} = \pi \cdot (.2488ft) \cdot (2910.62rpm) \rightarrow V_{av} = 2275.73\frac{ft}{min}$$

Tangential Force Calculation:

$$F_t = (33000 \cdot 427.55hp)/(2275.73\frac{ft}{min}) \rightarrow F_t = 6199.83lbs$$

The radial and axial forces can be found using the following equations:

$$F_n = F \sin \phi = F_t \tan \phi \qquad \text{(f)}$$

$$F_a = F_n \sin \gamma = F_t \tan \phi \sin \gamma \qquad \text{(16.22)}$$

$$F_r = F_n \cos \gamma = F_t \tan \phi \cos \gamma \qquad \text{(16.23)}$$

Normal Force Calculation:

$$F_n = (6199.83lbs)tan(20) \rightarrow F_n = 2256.55lbs$$

Axial and Radial Force Calculation:

$$F_a = (2256.55lbs)sin(45) \rightarrow F_a = 1595.62lbs$$

$$F_r = (2256.55lbs)cos(45) \rightarrow F_r = 1595.62lbs$$

x_1 *Calculation:*

It is assumed the bearing is right next to the gear

$$x_1 = (\frac{2.986in}{2})tan(45) \rightarrow x_1 = 1.493in$$

Finding Reaction Forces:

$$F_{netx} = A_x + B_x - F_a \rightarrow 0 = 0 + B_x - 1595.62lbs$$

$B_x = 1595.62lbs$

$\Sigma M_{Az} = 0 = (2.362in)B_y + (2.362in + 1.493in)(1595.62lbs) - (\frac{2.986in}{2}$

$B_y = \ - 1595.62lbs$

$F_{nety} = A_y + B_y + F_r \rightarrow 0 = A_y + (-\ 1595.62lbs) + 1595.62lbs$

$\rightarrow A_y = 0lbs$

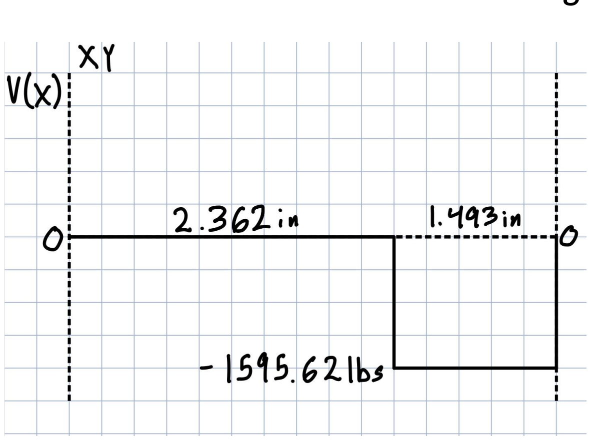
$\Sigma M_{Ay} = 0 = (2.362in)B_z + (2.362in + 1.493in)(6199.83lbs)$

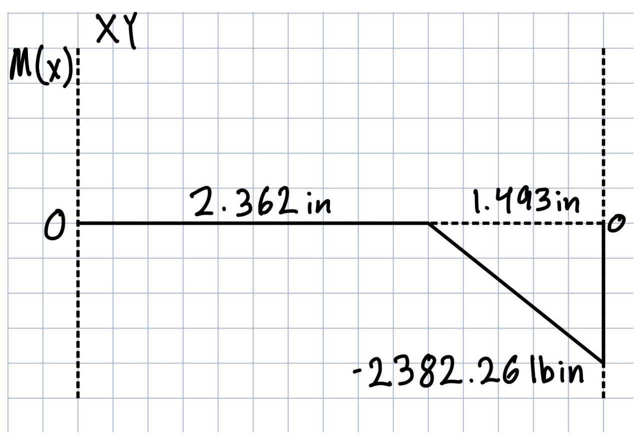
$B_z =- 10118.68lbs$

$F_{netz} = A_z + B_z + F_t \rightarrow 0 = A_z + (-\ 10118.68lbs) + 6199.83lbs$

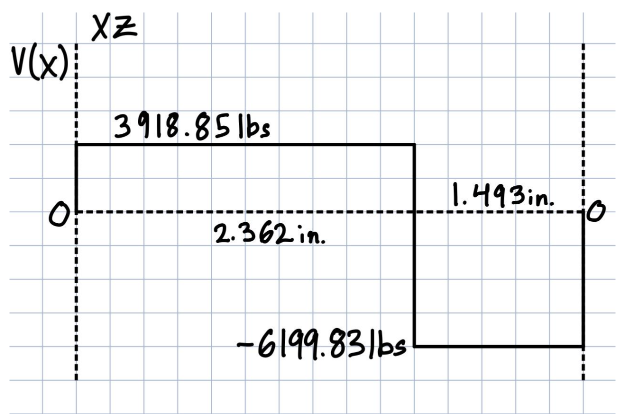
$A_z = 3918.85lbs$

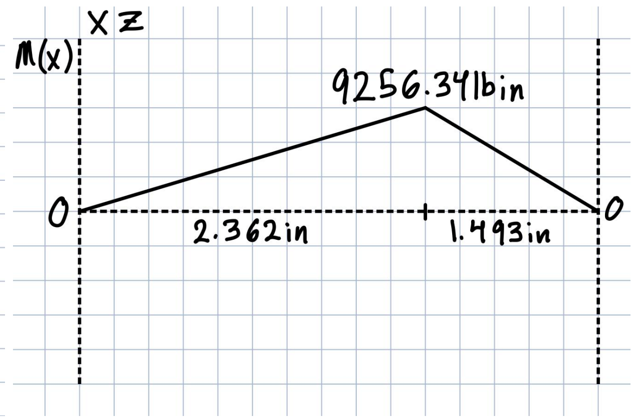
Horizontal Shear and Moment Diagrams:



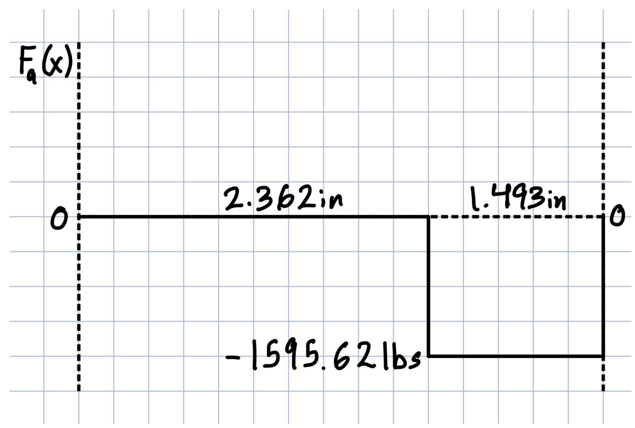


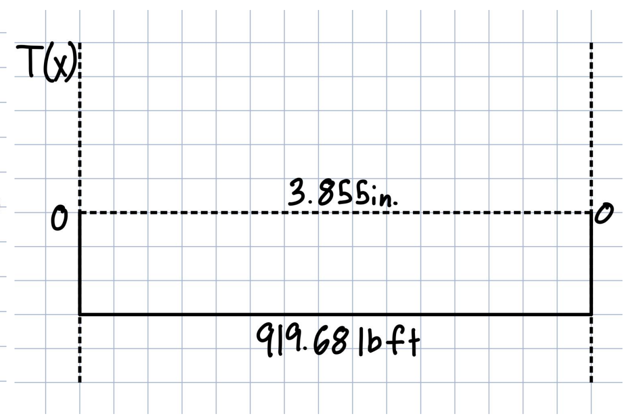
Vertical Shear and Moment Diagrams:





Axial Load and Torque Diagrams:





Loads on Bearings A and B:

$$Ar \;=\; \sqrt{(0lbs)^2 + \;(3918.85lbs)^2} = \;3,918.85\; lbs$$

$$Br \;=\; \sqrt{(-\;1595.62lbs)^2 + \;(-\;10118.68lbs)^2} = \;10,243.71\; lbs$$

$$Bt \;=\; 1595.62\; lbs\; (thrust)$$

At the most critical section (just to the left of the gear), shear force is given by the equation:

$$\tau_m = \frac{16T}{\pi d^3}K_f = \frac{16(11036.16lb \cdot in)}{\pi \cdot (2in)^3} \cdot 1.2 \text{ = 8431.0052 lbs}$$

The shaft we chose on Mcmaster (8929T32) has a 2in diameter, made from 1026 carbon steel. S_y = 75,000 psi; : S_u = 95,000 psi; K_f = 1.2 (torsion); K_f = 1.3 (bending), K_f = 1.3 (axial).

$$\sigma_{a,m}(\textbf{axial mean stress}) = \frac{P}{A}K_f = \frac{-1595.62lbs}{\frac{1}{4}\pi \cdot (2in)^2} \cdot 1.3 = -660.27 \text{ psi}$$

$$\sigma_{b,a}(\textbf{bending alt. stress}) = \frac{32M}{\pi d^3}K_f$$

$$= \frac{32\left(\sqrt{(-6199.83lb \cdot in)^2 + (9256.34lb \cdot in)^2}\right)}{\pi \cdot (2lb)^3} = 14184.92 \text{ psi}$$

From Table 8.2, we find that the equivalent alternating bending stress $\sigma_{e,a}$, is calculated from the distortion energy theory as being equivalent to the combination of existing alternating stresses:

$$\sigma_{e,a} = \sqrt{\sigma_{b,a}{}^2 + 0} = \sigma_{b,a} = 14184.92 \text{ psi}$$

Also from Table 8.2, we find that the equivalent mean bending stress σ_{em} , is taken as the maximum principle stress resulting from the superposition of all existing static mean stresses:

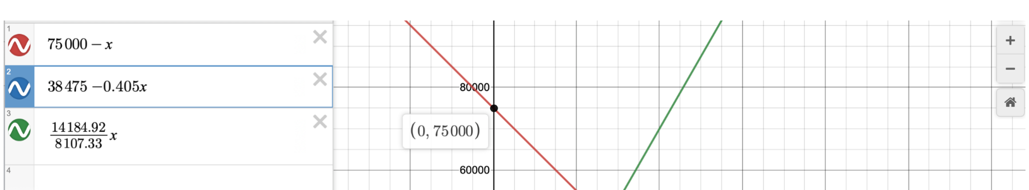
$$\sigma_{em} = \frac{\sigma_m}{2} + \sqrt{\tau_m{}^2 + \left(\frac{\sigma_m}{2}\right)^2} =$$

$$-\frac{660.27psi}{2} + \sqrt{\left(-\frac{660.27psi}{2}\right)^2 + (8431.0052lbs)^2} = 8107.33 \text{ psi}$$

From figure 8.5:

$$S_n = S_n' C_L C_G C_S$$ = (95,000psi/2)*1*0.9*0.9 = 38,474 psi

From this information we can graph the Sn and Su curves to find the safety factor of the shaft:



S.F. = $\frac{31,242.996psi}{14,184.92psi}$ = 2.2

Shaft B + C Critical Speed:

$w = A\rho = (\Pi/4) * (2in^2 - 1.5in^2) * 0.2839lb/in = 0.3902\ lb/in$

From appendix B-1 for a hollow shaft of a uniform weight distribution:

$I = (\Pi/64)$ * (d_o^4-d_i^4) = $(\Pi/64)$ * ($2in^4$-$1.5in^4$) = 0.53689 in^4

From appendix D-2 for a hollow shaft of a uniform weight distribution:

$\delta_{st} = \frac{5\ wL^4}{384EI}$ where E = 30 * 10^6 psi (appendix C-1)

$\delta_{st} = \frac{\left(5 \cdot \left(0.3902\left(\frac{lbm}{in}\right)\right) \cdot (15in)^4\right)}{384 \cdot 30 \cdot 10^6 \cdot \left(0.53689in^4\right)}$ = 1.5969*10^{-5} rpm

Using the equation from figure 17.5(c) to find the critical shaft speed:

$n_c \approx \sqrt{\frac{5g}{4\delta_{st}}} = \sqrt{\frac{\left(5 \cdot 386.088583\,\frac{in.}{s^2}\right)}{4 \cdot 1.59692497 \times 10^{-5} in.}}$ = 5497.39 rpm

This is significantly less than the maximum RPM that shafts B and C will ever spin at (2910.62 rpm).

Bearing Analysis (Done by Moustafa Checked by Omar)

The LM104949 bearings were analyzed using equations 14.3 and 14.5a front the textbook. Values are from the shaft analysisThe reliability factor was taken from table 14.13 ($K_r = .814$ for 99% reliability), the application factor was taken from table 14.3 ($K_a = 1.5$ for moderate shock), and the rated of 23400lbs was taken from the bearings manufacturer specifications. The required life is one million cycles for this differential.

$$\text{For } 0 < F_t/F_r < 0.35, \quad F_e = F_r$$
$$\text{For } 0.35 < F_t/F_r < 10, \quad F_e = F_r\left[1 + 1.115\left(\frac{F_t}{F_r} - 0.35\right)\right]\}$$
$$\text{For } F_t/F_r > 10, \quad F_e = 1.176F_t \qquad (14.3)^2$$

$$L = K_r L_R (C/F_e K_a)^{3.33} \qquad (14.5a)$$

Force Equivalent Calculation:

$$F_e = (1595.62lbs)[1 + 1.115(\frac{10118.68lbs}{1595.62lbs} - .35)] \rightarrow F_e = 7885.73lbs$$

Life Calculation:

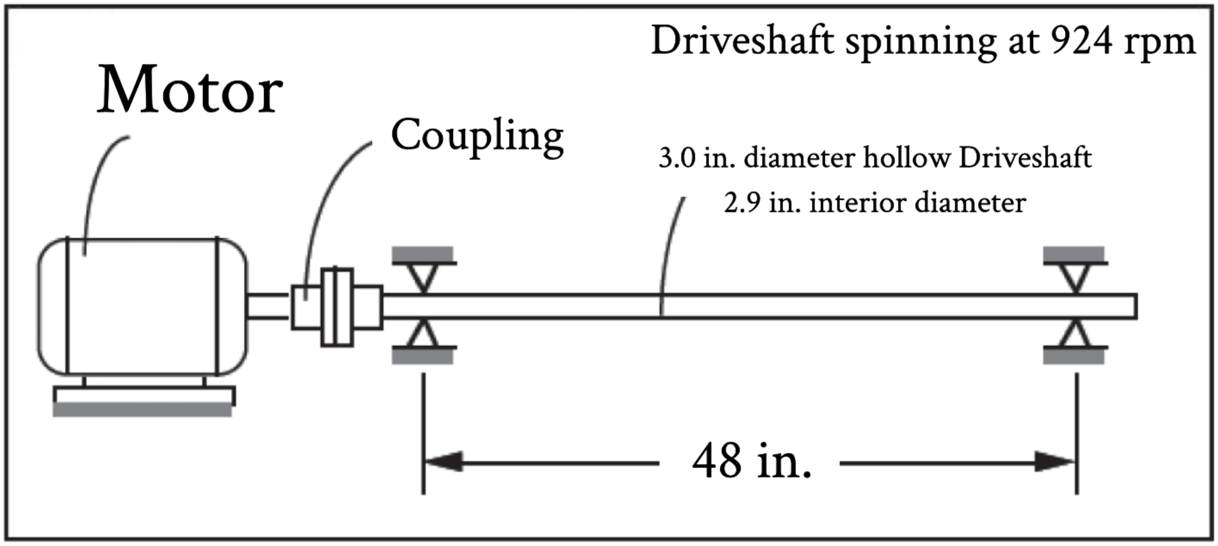
$$L = (.814)(10^6 cycles)(\frac{23400lbs}{(7885.73lbs)(1.5)})^{3.33} \rightarrow L = 7 \times 10^6 cycles$$

The bearings have a life cycle of seven times more than the required 1 million cycles. The bearings will not fail for the expected life of the differential.

Conclusions:

The differential needs some redesign to be successful. A new crown gear must be chosen that can handle this design.

HW Problem:



When the 2021 BMW M3 Competition sedan is traveling at its top speed of 180 MPH, its driveshaft spins at 924 rpm. Given the information in the diagram above, determine the critical speed of the simply supported steel driveshaft. Will the driveshaft reach this critical speed when the BMW is traveling at its maximum velocity? If it does, what are some possible factors of the shaft that could be changed to increase its critical speed?

HW Solution:

Assumptions:

1) Bearing friction is negligible
2) Bearings are perfectly aligned
3) The shaft remains linearly elastic

Analysis:

For a simply supported shaft of uniform weight:

w = weight per unit length $= A\varrho = \frac{\pi d^2}{4}\varrho$ where $\varrho = 0.28\frac{\text{lb}}{\text{in.}^3}$ for steel

For our problem :

w = $\frac{\pi}{4}\left((3in)^2 - (2.9in)^2\right) \cdot 0.28\frac{lb}{in^3}$ = 0.1297 lb/in

From Appendix D-2 in the textbook:

δ_{st} = maximum deflection of the driveshaft = $\frac{5\ wL^4}{384EI}$

Where: I = $\frac{\pi d^4}{64}$ = $\frac{\pi}{64}\left((3in)^4 - (2.9in)^4\right)$ = 0.5042 in^4 (From Appendix B-1)

And: E = 30 x 10^6 psi

Therefore: δ_{st} = $\frac{\left(5 \cdot 0.1297\frac{lb}{in} \cdot (48in)^4\right)}{384 \cdot \left(30 \cdot 10^6 psi\right) \cdot 0.5042207122\ in^4}$ =5.9265 x 10^{-4} in.

Finally, we can determine the critical speed of the driveshaft using the equation for a uniform shaft from figure 17.5c in the textbook.

$$n_c \approx \sqrt{\frac{5g}{4\delta_{st}}}$$

Where g is measured in in/s^2 = 386.09 in/s^2

N_c = $\sqrt{\frac{\left(5 \cdot 386.08858\frac{in}{s^2}\right)}{4 \cdot \left(5.9265475 \times 10^{-4}\ in\right)}}$ = 902.23 rpm

Since our driveshaft is spinning at 924 rpm when the car is at its maximum velocity, the shaft **will** reach the critical speed of 902.23 rpm. This can potentially produce vibrations that result in driveshaft failure.

To avoid this problem, there are three design changes we can make to the driveshaft. First, and most simply, we can make the driveshaft out of lighter material with a lower density. For example, if we changed the shaft material from steel to aluminum (which has a density of about 0.10 lb/in^3 compared to the density of steel at 0.28 lb/in^3), the driveshaft critical speed increases to 1509.7 rpm:

$$\sqrt{\frac{\left(5 \cdot 386.08858 \frac{in}{s^2}\right)}{4 \cdot \left(\left(\frac{\left(5\left(\left(\frac{\pi}{4}\left((3in)^2 - (2.9in)^2\right) \cdot 0.10 \frac{lb}{in^3}\right)\right) \cdot (48in)^4\right)}{384 \cdot 30 \cdot 10^6 \cdot \left(\frac{\pi}{64}\left((3in)^4 - (2.9in)^4\right)\right)}\right)\right)}} = 1509.7 \text{ rpm}$$

Although this increases the critical speed of the shaft, it's important to note that a lighter material such as aluminum has a lower yield and ultimate strength of the material, making it more likely to fail under applied loads. For this reason, the majority of driveshafts in modern vehicles are made from steel, although some use an aluminum shaft with weld yokes welded on either end.

Second, we can adjust the length of the driveshaft to make it shorter. For example, by decreasing the length of the driveshaft by 5 in. to 43in. Total length, the driveshaft critical speed increases to 1124.3 rpm:

$$\sqrt{\frac{\left(5 \cdot 386.08858 \frac{in}{s^2}\right)}{4 \cdot \left(\left(\frac{\left(5\left(\left(\frac{\pi}{4}\left((3in)^2 - (2.9in)^2\right) \cdot 0.28 \frac{lb}{in^3}\right)\right) \cdot (43in)^4\right)}{384 \cdot 30 \cdot 10^6 \cdot \left(\frac{\pi}{64}\left((3in)^4 - (2.9in)^4\right)\right)}\right)\right)}} = 1124.3 \text{ rpm}$$

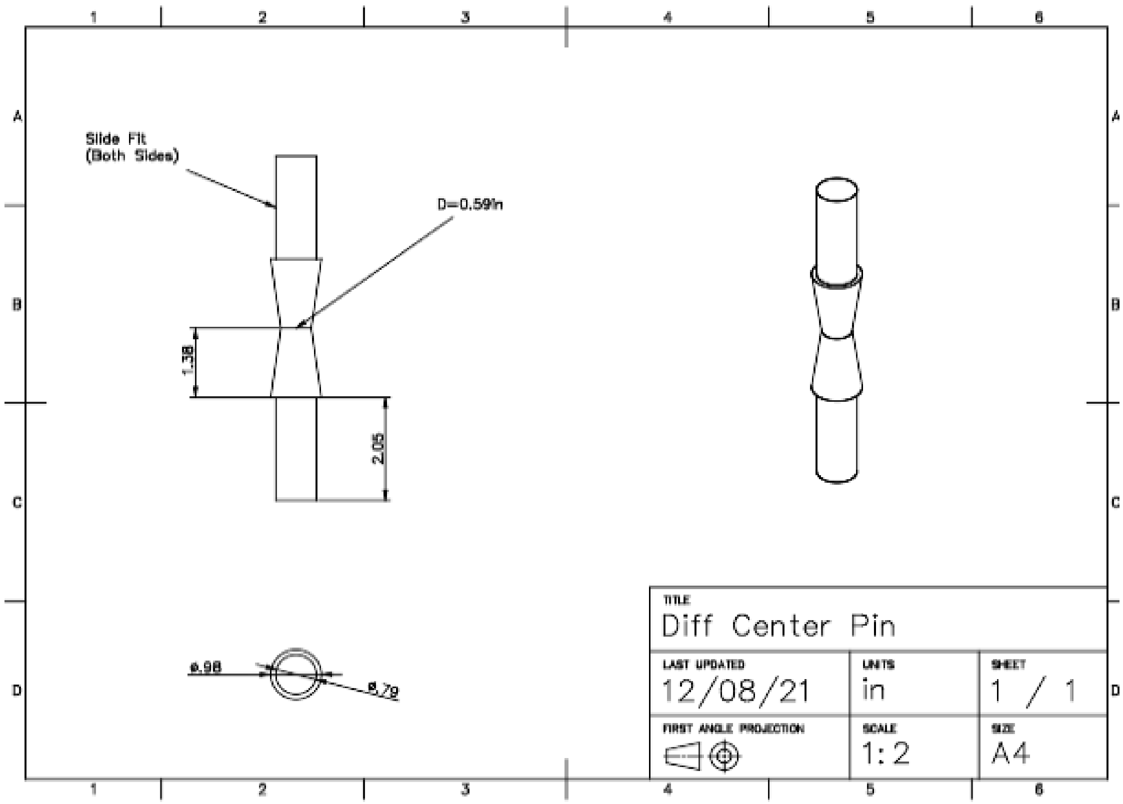
Finally, we can choose a shaft with a larger diameter. For example, by increasing the shaft diameter to 4 in. and the interior shaft diameter to 3.9 in., the driveshaft critical speed increases to 1208.0 rpm:

$$\sqrt{\frac{\left(5 \cdot 386.08858 \frac{in}{s^2}\right)}{4 \cdot \left(\left(\frac{\left(5\left(\left(\frac{\pi}{4}\left((4in)^2 - (3.9in)^2\right) \cdot 0.28 \frac{lb}{in^3}\right)\right) \cdot (48in)^4\right)}{384 \cdot 30 \cdot 10^6 \cdot \left(\frac{\pi}{64}\left((4in)^4 - (3.9in)^4\right)\right)}\right)\right)}} = 1208.0 \text{ rpm}$$

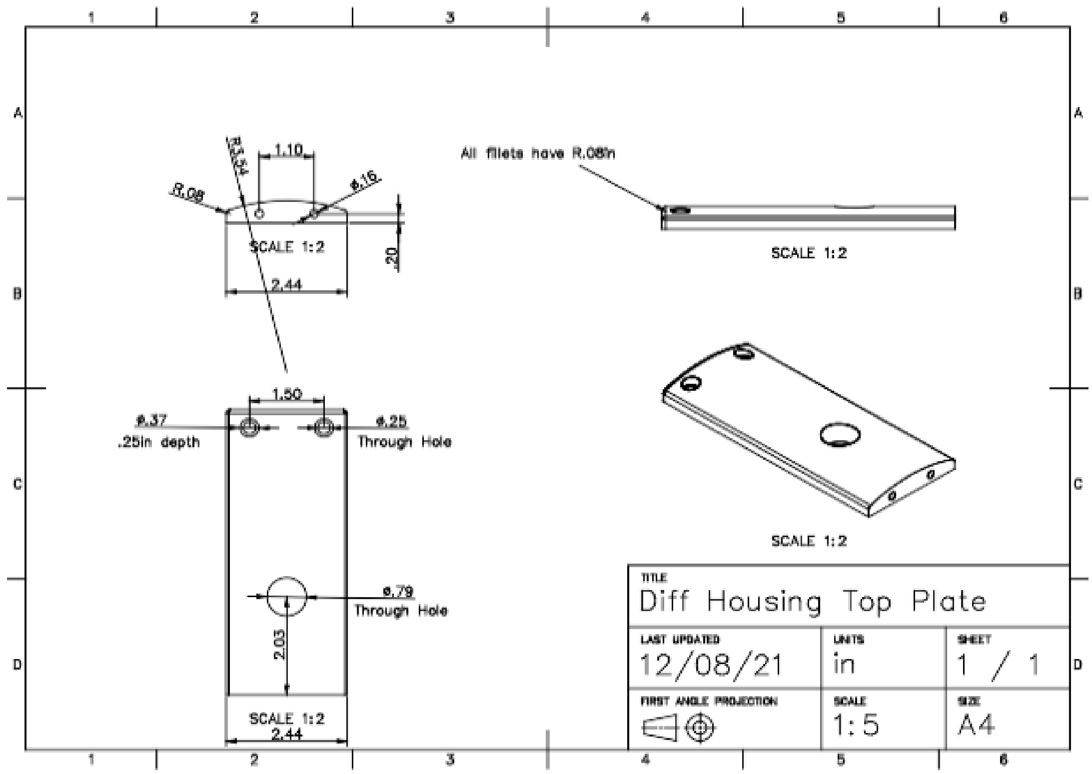
Extra Credit:

Engineering drawings were made of the housing assembly for manufacturing.

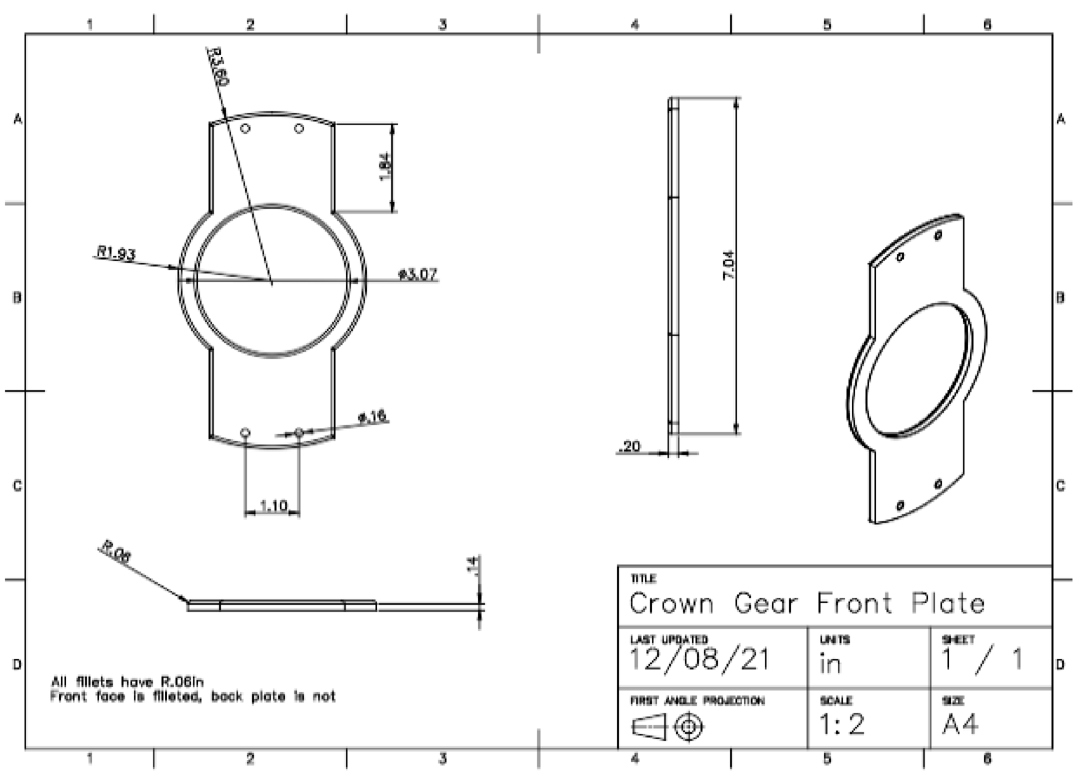
Center Pin:



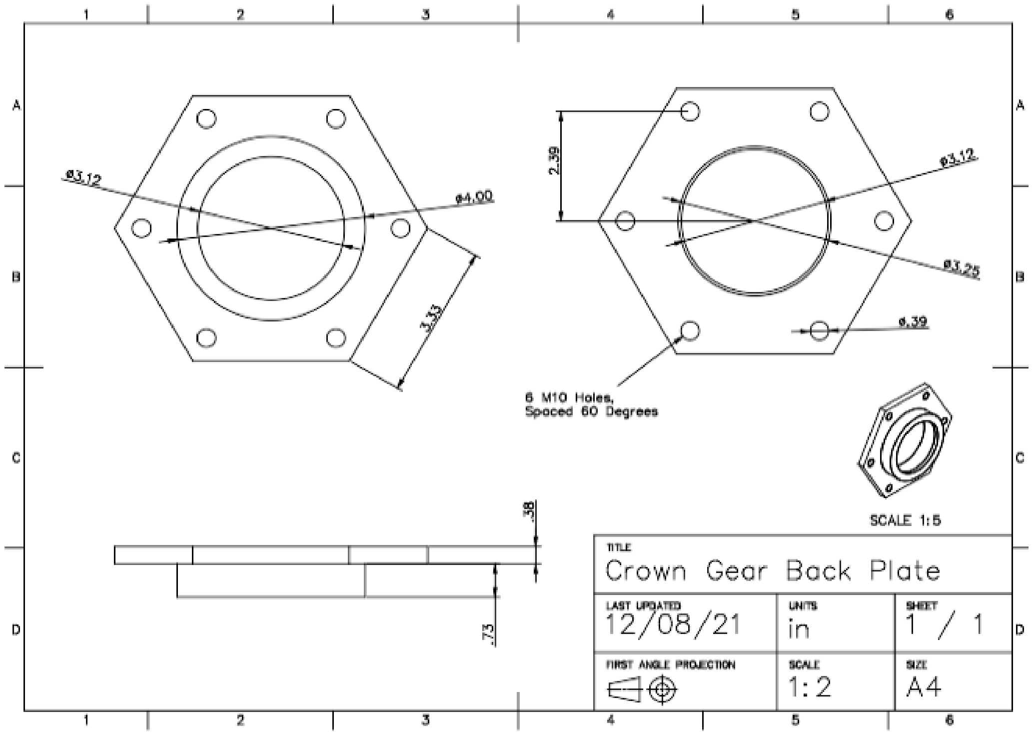
Housing Tope Plate:



Crown Gear Front Plate:



Crown Gear Back Plate:



Manufacturers:

Pinion gear - https://www.khkgears.us/catalog/product/MBSA6-1545L
Crown Gear - https://www.khkgears.us/catalog/product/MBSA6-4515R
Miter Gears - https://www.khkgears.us/catalog/product/MM3-30
Taper-Roller Bearings - https://www.mcmaster.com/5709K91/
2.5in Black-Oxide Steel Head Screw - https://www.mcmaster.com/90044A514/
1in Black-Oxide Steel Head Screw - https://www.mcmaster.com/91251A442/
Steel Button Head Screw - https://www.mcmaster.com/97763A455/
1045 Carbon Steel Rod - https://www.mcmaster.com/8924K78/

Sources:

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Juvinall, Robert, and Kurt Marshek. *Fundamentals of Machine Component Design*. 7th ed., Wiley, 2020.

"M3 Sedan Pricing & Features: BMW USA." *M3 Sedan Pricing & Features | BMW USA*, https://www.bmwusa.com/vehicles/m-models/m3-sedan/pricing-features.html.

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"Turbo Tech: Crank vs Wheel Horsepower and Why It Matters for Your Turbo." *Garrett Motion*, 30 Apr. 2020, https://www.garrettmotion.com/news/newsroom/article/turbo-tech-crank-vs-wheel-horsepower-and-why-it-matters-for-your-turbo/.

Who did What:

Omar drew the preliminary and final drawings for the differential, worked on the load calculations, design and redesign, part selection, gear analysis, bill of materials, and CAD model of the differential. He also checked the bearing analysis, helped record the video presentation of the project, and made the engineering drawings for the project extra credit. In the final report, he worked on the explanation of loads, load calculations, design, redesign, part selection, bill of materials, and gear analysis sections.

Moustafa worked on the part selection, design, and bearing analysis, and also helped to record the video presentation of the project. In the final report, he worked on the part selection and bearing analysis sections. It should be noted that Moustafa was forced to miss a significant section of the semester through illness and time spent in the hospital, but was notably willing and actively looking to contribute as much as possible to the project whenever he was in good health.

Oscar worked on the design and redesign, load calculations, part selection, and shaft analysis. He also checked the bearing analysis, helped to record the video presentation of the project, edited the video presentation of the project, and created the homework problem on shaft analysis. In the final report, he worked on the abstract, part selection, shaft analysis, homework problem, who did what, lessons learned, and guidance for next year's class sections.

Lessons Learned:

First and foremost, make sure there are available parts for your design before you begin calculations. We had to put significant time and energy into redesigning and recalculating the different measurements of our differential because we were not able to find a large enough gear/pinion set with a gear ratio of 3.15 exactly, and had to use a set with a 3.0 gear ratio. Second, it's better to take the time to look into the complexity of your project idea before you start it; even the more simplistic open differential (compared to the limited slip differential used in the real BMW model) proved to be quite a complicated machine to design, and perhaps in hindsight we might have chosen something less taxing. Finally, it's always beneficial to set your deadline a day earlier than the actual due date; we really came down to the wire in completing our video presentation specifically.

Guidance for next year's Class:

- Start working on the design and analysis as soon as possible. Although you won't learn everything you need to know to complete the project until later in the semester, we would say the most efficient and painless way to complete the project on time would be to begin working on the part analysis as you learn them in class. Even if this was just setting up the problem for later it would really pay off later in the semester.

- The textbook is a great source of information for completing the different analysis; it has an example for pretty much every type of problem for each type of analysis.

- Reach out to the professor and TA about any questions you have that made the project a lot easier for us.